



## 2016 Online Competition

### Guidelines

Student teams will have a total of **one week** to complete the exam from start to finish. There are two separate sections for this examination: one section on Laser and Plasma Physics, and one section on Entropy and Statistical Mechanics. Teams may complete both sections or choose to complete only one, as is specified in the grading explanation below. We recommend that teams set aside approximately 20+ hours to allow enough time for successful completion. Please refer to the submission explanation below for details on both formatting and the submission process.

### Grading

Students are encouraged to work on as much of both sections of the exam as possible. However, teams **may choose to submit solutions for only one of the two sections** if they desire. The two sections will be graded separately, and may not necessarily be worth the same amount of points. The award structure will be as follows:

1. Awards will be given to the four teams with the **highest score in each section** (an award for first place, second place, third place, and fourth place). One team can win an award for both sections, such as second place in Laser and Plasma Physics and fourth place in Entropy and Statistical Mechanics. Therefore teams are encouraged to attempt solutions for both sections of the competition.
2. We will additionally award one **overall award to the highest scoring team** on the entire competition. A team which wins this overall award can still receive one of the top four awards for each individual section. The team that wins this overall award will most likely have completed both sections of the exam. It will be at the judges' discretion to choose the overall award for the best submission.
3. **Special awards** will also be given for honorable mentions, the most elegant solution, and the most creative solution.

### Collaboration Policy and Resources

Students participating in the competition may only correspond with other members of their team. No other human correspondence is allowed, including: mentors, teachers, professors, and other students. In general, participating students are barred from posting content or asking questions related to the exam on the internet (except where specified below). Students are, however, allowed to use the following resources:

- **Online:** Teams may use any information they find useful on the Internet. However, under no circumstances may they actively post content or ask questions about the exam.
- **Piazza page:** Teams are encouraged to create an account on Piazza and register in the class at the following URL: [http://piazza.com/princeton\\_university\\_physics\\_competition/fall2016/pupc2016](http://piazza.com/princeton_university_physics_competition/fall2016/pupc2016). The access code is: **pupc2016**. This resource can be used by teams to ask questions about the content of the exam. Please do not post any of your solutions, partial or complete, when asking questions on Piazza.

- **Published Materials:** Teams may take advantage of any published material, both printed or online.
- **Computational:** Teams may use any computational resources they might find helpful, such as Wolfram Alpha/Mathematica, Matlab, Excel, or lower level programming languages (C++, Java, Python, etc). For some sections, the use of computational resources is highly advised.

## Citations

All student submissions with outside material must include numbered citations. We do not prefer any style of citation in particular. Students may find the following guide useful in learning when to cite sourced material: <http://www.princeton.edu/pr/pub/integrity/pages/cite/>.

## Submission

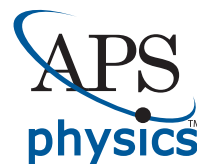
All submissions, regardless of formatting, should include a **cover page listing the title of their work, the date, and signatures of all team participants**. The work must be submitted as **one single PDF document with the “.pdf.” extension**. All other formatting decisions are delegated to the teams themselves. No one style is favored over another. That being said, we recommend that teams use a typesetting language (e.g.,  $\text{\LaTeX}$ ) or a word-processing program (e.g. Microsoft Word, Pages). Handwritten solutions are allowed. **Note: we reserve the right to refuse grading of any portion of a team’s submission in the case that the writing or solution is illegible.**

Teams must submit their Online Part solutions by e-mailing [pupc@princeton.edu](mailto:pupc@princeton.edu) by 11:59 am (noon) Eastern Time (UTC-5) on Saturday, November 19, 2016. Teams will not be able to submit their solutions to the Online Part at any later time. Any team member may send the submission. The title of the submission e-mail should be formatted as “SUBMISSION - Team Name”. Note: all teams may make multiple submissions; however, we will only grade the most recent submission submitted before the deadline. **Teams will receive confirmation once their submission has been received within at most two days.** In the case of extraordinary circumstances, please contact us as soon as possible.

## Sponsors



## Collaborators



# Laser and Plasma Physics Section

The subject of this document is a link between plasma physics and laser physics. More specifically, we will see how we can use plasmas to obtain a very strong laser amplification. We will also try to understand the theory of laser amplification, which is roughly the process of obtaining a very high laser intensity from a lower intensity laser beam. Laser amplification technology has become highly advanced since the 1960's through methods such as Mode-locking, Q-switching and Chirped Pulse Amplification. These methods have allowed huge improvements in laser intensity, with beams reaching intensities of up to  $10^{20}$  W/cm<sup>2</sup>, in a focused beam. However, these methods are limited by the material which is used to amplify the laser beam. Solid state lasers use amplification medium crystals, such as Titanium-sapphire crystals, or the more common neodymium-doped yttrium aluminium garnet, known as YAG. These materials deteriorate at very high intensities due to the resulting high temperature, so they are limited to a certain maximum laser beam intensity.

In this paper, you will explore a new idea to enhance laser amplification by using a process called Raman-Backscattering. Raman-Backscattering is the process of having a photon hit an atom, excite it to a certain state, and then have another photon be re-emitted at a different frequency from the initial frequency in the direction of the initial incoming atom. This process can roughly turn a long high energy laser pulse into a short, very high intensity laser pulse. (We will actually use two laser beams to obtain this result later in this document.) Instead of a normal amplification medium you will use plasma, which is already broken up into ions and electrons, so it cannot deteriorate further in a high intensity laser. You will work on some small problems to get accustomed to the ideas of oscillations in plasmas and to the idea of Raman Scattering. Finally, you will implement a computer simulation of a method of amplifying laser beams.

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**Learning goals of this topic:** The goal of this topic is to let students engage with a current topic in plasma physics research and to develop an intuition for how plasmas and lasers work. The most important goal for the student is to learn how to simulate a physical process and to learn to interpret the results of what they simulate.

**Topic format:** This document consists of a preparation section with questions that will help the student start thinking about the notions important for the simulation. This section of the competition also contains a simulation section, to be performed in Matlab (a coding environment that can be downloaded as a free trial version online. See below.). The purpose of the simulation is to teach students about advanced coding

concepts. Even more so, it is to make students think about how to transform differential equations into code to solve physical problems. It is also about relating the code you write and the simulation results with physical arguments. Even if you can't solve all the exercises in the theory part of this document, you can still attempt the simulation part, as numerical answers are provided for the parameters you are asked to obtain in the first part of this document. Note: you should submit the code you have written at the end of the competition along with your answers to other questions in this document.

**Example coding files provided to assist in simulation:** You are free to proceed with the Matlab simulation in however way you see fit as long as you address the questions brought up in the document below and ultimately simulate the physical process described below. This physical process is described in the last half of this document, but the exact implementation is up to the teams to decide.

To help with getting started performing the simulation, you are provided with a Matlab coding frame file entitled "PUPCcodeFrame.m". It contains hints and some initial code to help you stay organized as you follow the suggested simulation steps in this document. In addition, an example Matlab simulation is provided that simulates the movement of a ball under the influence of gravity, entitled "Ballexample.m". This simple model exists as a guideline to help you get started with the larger simulation, and is not an actual part of the competition. When you submit your solution, submit your code, the graphs you make, and the solution to the questions within this document.

## Additional reading materials and resources

You may find a few outside sets of information for this topic useful:

1. **Matlab Free Trial** A Matlab free trial can be downloaded at:

[https://www.mathworks.com/programs/trials/trial\\_request.html?s\\_tid=srchbrcm](https://www.mathworks.com/programs/trials/trial_request.html?s_tid=srchbrcm).

2. **Basics of Matlab** The language of Matlab is pretty simple and intuitive and you should not be intimidated by the syntax and details of the coding language. Even if you have no experience with coding in Matlab, you can search for solutions to coding problems online. A good basic resource is the PowerPoint presentation by Vince Adams and Syed Bilal Ul Haq, "Introduction to Matlab", which can be found online. The Matlab notions you actually need to be familiar with are basic matrices and vectors, "for" loops, "if" statements and "else" statements. Specific resources for these relevant syntax elements are:

-the colon ":" operator: <https://www.mathworks.com/help/matlab/ref/colon.html>.

-creating an array (matrix, or vector) of zeros: <https://www.mathworks.com/help/matlab/ref/zeros.html>.

-using a "for" loop: <https://www.mathworks.com/help/matlab/ref/for.html>.

-using the "if" statement: <https://www.mathworks.com/help/matlab/ref/if.html>.

-arrays in general: [https://www.mathworks.com/help/matlab/learn\\_matlab/matrices-and-arrays.html](https://www.mathworks.com/help/matlab/learn_matlab/matrices-and-arrays.html).

Even though all the plotting functions that you will probably need have been given to you in the code frame, you may also find the following documentation on plotting functions useful: <https://www.mathworks.com/help/matlab/ref/plot.html>.

3. **General Matlab Documentation** These websites are a good starting point for solving more general coding problems in Matlab:

<https://www.mathworks.com/help/matlab/>

or

<https://www.mathworks.com/help/matlab/getting-started-with-matlab.html>.

4. **Euler Approximation method** The Euler Approximation method is useful to understand to interpret the simulation:

[https://en.wikipedia.org/wiki/Euler\\_method](https://en.wikipedia.org/wiki/Euler_method).

5. **Vector calculus** It would be useful to understand vector calculus concepts such as the gradient, divergence, and curl for some problems:

[https://en.wikipedia.org/wiki/Vector\\_calculus\\_identities](https://en.wikipedia.org/wiki/Vector_calculus_identities).

# 1 Introduction

We will start by working with some simple models of plasma oscillations as they will be helpful in seeing how plasma behaves when it interacts with electric fields. We will then discuss some about how magnetic potential relates to laser intensity and we will do a simple model of Raman Scattering.

## 2 Theory

### 2.1 Plasma Oscillation

A neutral plasma is defined as a collection of particles: ions and electrons. Ions have positive charge  $+e > 0$  and electrons have negative charge  $-e < 0$ . They have equal number densities  $n_e$ . In the first part of the problem we shall model the small oscillations that happen in cold plasmas (plasmas with negligible thermal motion) because of small disturbances in the arrangement of the electrons and ions.

Imagine a collection of fixed ions. We can consider the ions fixed as they have a much larger mass than the electrons and move very little compared to the electrons. The freely moving electrons have equal density with the ions and fill all the space. Set a 3 axes coordinate system  $x, y, z$ . Consider the  $yz$  plane and  $x$  the axis perpendicular to it. Consider all the electrons with  $x > 0$  to be displaced a distance  $d$  in the positive direction, and all the electrons for which  $x < 0$  to be displaced a distance  $d$  in the negative direction. (You can ignore transient effects due to this initial displacement and take it as a given.) There will be an excess positive net charge due to the ions in the infinite slab centered at  $x=0$  with width  $2d$ . This charge creates an electric field, which pulls the electrons back towards the origin. This force, which pulls the electrons, creates an oscillatory motion.

Question: Describe the movement of an electron due to this electric field and assume that the ions are stationary. Find the oscillation frequency  $\omega_{pl}$ . Note: consider electrons directly at the border of the slab. Give your answer in terms of  $n_e$ , electron number density,  $m_e$ , mass of the electron, and  $e$ , charge of the electron.

### 2.2 The Langmuir Wave: a Warm Model for the Plasma Oscillation

The frequency we have calculated in the previous part was a very simple model for plasma oscillations. To obtain a better model we shall start from the equations that model the motion a fluid and analyze a small piece of fluid, made of particles with density  $n$  and of volume  $dV$ . The equation of motion for this fluid is:

$$\rho_m \frac{d\vec{v}}{dt} = -\vec{\nabla}p + \vec{f} \quad (1)$$

where  $\rho_m$  is the mass density of the fluid,  $v$  is the speed,  $p$  is the pressure, and  $\vec{f}$  is the force acting on it.

This expression is derived from the formula  $\vec{F} = m\vec{a}$ , with  $m\vec{a}$  corresponding to  $\rho_m \frac{d\vec{v}}{dt}$ ,  $\vec{f} = \frac{dF_{ext}}{dV}$  corresponding to the external force on a small volume  $dV$  divided by  $dV$ , the term  $\vec{\nabla}p$  represents force per volume  $dV$  and arises from the gradient (difference) of pressure on the edges of a small volume. We notice that  $-\vec{\nabla}p$  points from higher to lower pressure. Another useful expression is the conservation of charge equation, which says that:

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho\vec{v}) = 0. \quad (2)$$

This equation says that the growth with respect to time of charge density  $\rho$  in a small volume  $dV$  plus the divergence of charge density current that leaves a certain volume is equal to 0. Note that  $\rho$  now refers to a charge density, meaning a charge multiplied by a density. What this expression means is simply that the charge that leaves a volume is equal to the change in the charge in that volume.

Let's assume first that density variation and velocity variation are small and that we can neglect second order and higher terms in a Taylor expansion around typical density and typical speed. Consider a 1 dimensional model, only with respect to an  $x$  axis. We consider small variations in the fluid, with  $n = n_0 + n'$ , where  $n_0$  is a constant density term when there is no movement in the fluid and  $n'(x)$  is a small deviation from  $n_0$ . We can also write the speed as  $v = v_0 + v'$ , where  $v_0 = 0$  is the speed when there is no movement

of particles in the fluid, and  $v'$  is a small deviation from  $v_0 = 0$ . Let us say that the particles that form the fluid are charged and consider that we have the charge conservation formula derived from (2)

$$e \frac{\partial n}{\partial t} + \frac{\partial(nev)}{\partial x} = 0 \quad (3)$$

where  $n$  is electron number density, and  $e < 0$  is the electron charge. Also in the more explicit form of the equation of motion, equation (1) we must replace  $p$  with  $nE_{kin}$ , where  $E_{kin}$  is the kinetic energy of a particle from the fluid. This is just the expression for pressure in a gas. Therefore we have

$$nm \frac{\partial v}{\partial t} = - \frac{\partial n}{\partial x} E_{kin} + f \quad (4)$$

and we drop vector notation as we work only on the  $x$  axis. In the following calculation you will use equations (3) and (4). We can neglect terms of the form  $\frac{dn'}{dt}v'$  and  $\frac{dv'}{dt}n'$  as they are second order terms. You can also neglect  $\frac{\partial n'}{\partial x}n'$ ,  $\frac{\partial v'}{\partial x}n'$ ,  $\frac{\partial n'}{\partial x}v'$ . Notice that  $\frac{f}{n_0m} = \frac{dF_{ext}}{dNm}$ , where  $\frac{dF_{ext}}{dN}$  is the “external force on one particle of the fluid”, meaning other forces except for the pressure term.

Question: Using all these approximations and the two equations (3) and (4), derive a differential equation in  $n'$ , containing the terms  $\frac{\partial^2 n'}{\partial t^2}$  and  $\frac{\partial^2 n'}{\partial x^2}$ . We will use this equation to get a very rough estimate of the Langmuir frequency.

We now notice that plasma, and other fluids such as gravitational attracted fluids are described by equations (3) and (4), with appropriate values for  $f$  or equivalently  $\frac{dF_{ext}}{dN}$  that depend on what type of fluid you are describing. The derivation for the fluids is valid for the plasma as well. We just need to put the correct formula for the external force created by Electromagnetic fields. We will derive the frequency of plasma oscillations using the fluid model for it. Use the fact that for an electric field  $\frac{\partial E}{\partial x} = \frac{\rho_e}{\epsilon_0}$  and that what creates the electric field in an oscillating plasma is the variation of density  $n'_e$ . Here,  $\rho_e$  is the charge density of the particles in the plasma. For the plasma we can write the density of the electrons as  $n_e = n_0 + n'_e$ , where  $n_0 = Zn_i$ , where  $Z$  is the atomic number of the ions, which have density  $n_i$ . These expressions mean that the charge of the ions and the charge of the electrons cancel at equilibrium, and the variations in  $n'_e$  are what matters. Pay attention to signs. Remember that the ions are immobile and that the electrons are mobile and think about what is the “External force on one particle (electron)” in case of a plasma.

Question: When you obtain a partial differential equation in  $n'_e$ , as required in the previous question, replace  $n'_e$  with  $Ae^{i(\omega t - kx)}$  which represents a plane wave. Do not be surprised if the equation may not look like the classical equation for a simple harmonic oscillator. Try to determine the dependence of the frequency  $\omega(k)$  on the wave vector  $k$ . Express the result for the frequency of oscillations in the Langmuir plasma as a function of  $E_{kin}$  (electron kinetic energy),  $k$ , and  $w_{pl}$  (the expression derived above) and other appropriate constants. Write an expression by replacing  $E_{kin}$  with  $v_{thm}$  (the thermal velocity of the electrons) and other constants. Depending on your modeling you might obtain slightly different results for the frequency  $\omega$ . If the derivation is correct, the results will be considered correct and valid.

## 2.3 Raman Scattering and Magnetic Potential

### 2.3.1 Raman scattering

Raman scattering is defined as an inelastic scattering of a photon with a certain material. In contrast to Rayleigh scattering, in which the scattered photon has the same energy as the incident photon, in Raman scattering the energy of the scattered photon is either greater or lower than that of the incident photon. This inelastic scattering is due to the interaction of the electromagnetic wave with the material.

To model this phenomenon, we consider a linearly oscillating electric field,  $E = E_0 \cos(\omega_0 t)$ , that interacts with the material and induces a dipole moment  $P = \alpha E$  in a molecule, where  $\alpha$  is the polarizability of the molecule. The induced dipole moment  $P$  is a source of electromagnetic radiation, creating the scattered light. The ability of the electric field to perturb the electron cloud of a molecule depends on the location of individual atoms. It follows that the polarizability  $\alpha$  is a function of instantaneous positions of atoms in a molecule.

In a molecule, atoms have specific vibrational modes with specific vibrational energies  $E_{vib}$ , with specific vibrational frequencies  $\nu_{vib}$ . The displacement  $dQ$  about the equilibrium position of atoms can be expressed

as  $dQ = Q_0 \cos(w_{vib}t)$ , where  $Q_0$  is the maximum displacement around the equilibrium position in a molecular bond. Let us assume that for our molecule, the maximum displacement  $Q_0$  is small compared to the bond length.

Question: Using first order approximations for the polarizability around the equilibrium position, try to obtain the three possible frequencies at which the molecule or substance can scatter light. The frequency greater than the initial frequency is called Anti-Stokes frequency, and the frequency lower than the initial frequency is called Stokes frequency. How will the intensities of the 3 different scattered waves, with three different frequencies be related to each other? What part of the scattered light (with which frequency) do you expect to be the most intense? The least intense? Please explain.

Observations: This is the classical view of the phenomena. One can also approach the problem from a more quantum mechanical point of view that atoms have a certain vibrational frequency  $\nu_{vib}$ . A photon can excite the electrons of a molecule from a mode with no vibration to an energetic level with frequency  $\nu_1$ . Next, the electrons can de-excite to the vibrational energy level of the molecule. From the de-excitation, a photon is emitted that has a frequency of  $\nu_1 - \nu_{vib}$  (for Stokes Frequency).

### 2.3.2 Magnetic potential

In this section we will relate the magnetic potential of a electromagnetic wave with the intensity of the wave.

The magnetic vector potential is a vector field  $\vec{A}$ , such that  $\vec{B} = \nabla \times \vec{A}$ , where  $\vec{B}$  is the magnetic field. As there are multiple fields  $\vec{A}$  that can satisfy this condition, one generally chooses the field  $\vec{A}$  that satisfies the propriety  $\text{div}(\vec{A}) = 0$ , so that  $\vec{A}$  has a definite value. Let us take a vector field of the form  $\vec{A}_0 = \hat{y} \frac{m_e c}{e} a_0(x, t) e^{i(k_0 x - w_0 t)}$  for a certain electromagnetic wave, where  $a_0(x, t)$  is a slowly varying envelope of the vector potential, and  $k_0 = \frac{2\pi}{\lambda_0}$  and  $w_0$  are the wave wavenumber, and the frequency, respectively.

Question. What is the value  $\vec{B}(x, t)$  that results from this expression for the magnetic potential? Derive an equation that describes  $a_0(x, t)$  as a function of the intensity  $I_0(x, t)$  of the electromagnetic wave (in the units of  $(\frac{W}{m^2})$ ),  $\lambda_0$ , and other constants. Remember that we have  $a_0(x, t)$ , a slowly variable envelope, so we can neglect the partial derivatives of  $a_0(x, t)$  with respect to time and space. For our purposes here, we will assume that the intensity of the wave is given by  $I_0 = cB^2/\mu_0$ . The constant  $\mu_0$  is the magnetic permeability constant of vacuum, while  $B(x, t)$  is the magnetic field strength. Average the square of the magnetic field to get the intensity.

We will need  $a_0(x, t)$  as a function of  $I_0(x, t)$  in order to calculate the values of the vector potentials for the pump and seed laser pulses in the simulation in the next part of this document.

## 3 Simulation: Laser Amplification in Plasma using Raman Backscattering

In this section we will do a computer simulation of a phenomenon using all the notions described so far. We will try to simulate a laser amplification technique which uses plasma to transfer energy from a long, intense laser pump pulse to a short, less intense laser seed pulse. In this process, plasma helps the transfer of energy from a pump laser pulse to a seed laser pulse, through a Langmuir wave. In the process, the seed grows in energy, the pulse time length gets shortened, leading to a very intense seed pulse and a depleted pump.

The seed with frequency  $w_2$  and pump with frequency  $w_1$  combine to create a new oscillating frequency  $w_1 - w_2$ , that sets the plasma in motion and creates a Langmuir wave. This Langmuir wave has a vibrational energy level with frequency  $w_2 - w_1$ . A pump photon with frequency  $w_1$  hits the plasma and excites it to the energetic level  $w_1$ , and because of the vibrational energy of the Langmuir wave it can de-excite to the energy level  $w_1 - w_2$ , a process which creates a scattered photon with frequency  $w_2$  in the direction of the seed pulse, leading to the amplification of the seed. This is a rough scheme of how Raman scattering and the Langmuir wave work in the process of amplification. Another way of thinking about the process is that a pump photon decays into a counter-propagating seed photon and Langmuir plasmon. This is a rather more accurate picture of what happens. This process is called Laser Amplification in Plasma with Raman Backscattering.



### 3.1 Description of the Code Frame File

To help with getting started performing the simulation, you are provided with a Matlab coding frame file entitled “PUPCcodeFrame.m”. Students should read through this code frame to get an idea of how the simulation can be set up. The code frame also contains some hints about how to approach the simulation. In addition, the following sections of this document go through various parts of the code frame and describe how to build up the entire simulation part by part. The code frame should be seen as a guideline. You may deviate from its structure if you still achieve a simulation of the desired physical process.

### 3.2 Description of the Simulation

Consider a tube, made of a channel of plasma of length  $l_{pl} = 0.004\text{m}$  and a small portion which we can consider to be filled with vacuum, of length  $l_{npl} = 0.001\text{m}$ , to to the right of the plasma channel. Say we send inside the tube two laser pulses from opposing directions, both travelling with speed  $c$  (the speed of light), one from the left, called pump, and one from the right, called seed. The seed pulse is a pulse with a short time duration  $t_s = 500 * 10^{-15}\text{s}$ , while the pump pulse is of longer duration,  $t_p = 15 * 10^{-12}\text{s}$ . Because the seed pulse has a shorter time, it has a shorter length. The seed has an energy of  $E_{seed} = 70 * 10^{-6}\text{J}$ , while the pump has an energy of  $E_{pump} = 0.5\text{J}$ . The wavelength of the pump is  $\lambda_p = 351 * 10^{-9}\text{m}$  and the wavelength of the seed is  $\lambda_s = 400 * 10^{-9}\text{m}$ . The diameter of both beams is  $150 * 10^{-6}\text{m}$ , and they are coaxial. The plasma is characterized by an electron density  $n_e = 1.1 * 10^{25}$  electrons per  $\text{m}^3$  and an electron mean kinetic energy  $E_e = 8\text{eV}$ . The two pulses start at time  $t = 0$  in the following way: The seed pulse starts just at the right end of the tube,  $x = l_{pl} + l_{npl}$ , and the pump pulse starts in a certain position so that, when they meet, they meet around the interface between the plasma and the vacuum.

Then the two pulses meet and interact with one another at a later time  $t$ . The ideal resonant conditions, in order for the pump photons to excite the electrons and, after that, to have the electrons de-excite to the vibrational energy level of the langmuir wave, are:

$$w_1 = w_2 + w_3 \quad (5)$$

where  $w_1, w_2, w_3$ , are the frequency of the pump, the seed and  $w_3$  and the Langmuir wave, respectively. We also have that

$$k_3 = k_1 + k_2 \quad (6)$$

where  $k_1, k_2, k_3$ , are the positive wavenumbers of the pump, seed and Langmuir wave respectively. Positive wavenumber means that it propagates to the right, and negative wavenumber means it propagates to the left. Look at equations (7), (8), (9) to understand why this is true. In our simulation, equation (6), will be exactly satisfied in the simulation we will do, while we will set the parameters not to satisfy (5).

In section 2 you have calculated the frequency of oscillation of the Langmuir Wave. A more precise calculation leads to the result  $w^2 = w_{pl}^2 + 3k^2 v_{thm}^2$ , with  $v_{thm} = (2E_e/m_e)^{1/2}$ .  $E_e = k_b T_e$  is the kinetic energy of an electron and  $k_b$  is Boltzman’s constant and  $T_e$  is electron temperature. From now on, we will use this result for the Langmuir wave frequency. Also, if you have not managed to get the result for section 2.3.2, you can use the formula for the envelope  $a_0(x, t) = .855 * 10^{-5} * \lambda_0 \sqrt{I_0(x, t)}$ .

We take the form of the magnetic potential for the pump and seed beam to be:

$$\vec{A}_1 = \hat{y} \frac{m_e c}{e} a_1(x, t) e^{i(k_1 x - w_1 t)} \quad (7)$$

for the pump

$$\vec{A}_2 = \hat{y} \frac{m_e c}{e} a_2(x, t) e^{i(-k_2 x - w_2 t)} \quad (8)$$

and for the seed. The electrostatic field of the Langmuir can also take the form

$$\vec{E}_3 = \hat{x} \frac{m_e}{e} \sqrt{w_1 * w_{pl}} a_3(x, t) e^{i(-k_3 x - w_3 t)} \quad (9)$$

with  $a_1(x, t)$ ,  $a_2(x, t)$  and  $a_3(x, t)$  are real (not imaginary) slowly varying envelopes. Notice that  $a_1, a_2, a_3$  are dimensionless quantities. These equations, with the appropriate approximations, simply mean that we

work with linearly polarized seed and pump pulses, meaning that the electric field (or magnetic) oscillate only in a direction.

As we can see from section 2.3.2,  $a_1(x, t)$  is proportional to  $\sqrt{I_1}$  of the pump beam,  $a_2(x, t)$  is proportional to  $\sqrt{I_2}$  of the seed beam,  $a_3(x, t)$  (from the electric field of the Langmuir wave) is also proportional to  $\sqrt{I_3}$ . This means that we can analyse what happens to the slowly varying envelopes  $a_1, a_2, a_3$ , and transform the results back to see what intensities we obtain. So we can think of  $a_1, a_2, a_3$  as some quantities that correspond to the intensities of the pump and seed pulses and the Langmuir wave respectively.

The modeling of this interaction, between the two laser waves and the Langmuir wave, is done with the help of Maxwell's equations. After doing many approximations the resulting three equations that describe the system are:

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial x}\right)a_1(x, t) = Ka_2(x, t)a_3(x, t) \quad (10)$$

$$\left(\frac{\partial}{\partial t} - c\frac{\partial}{\partial x}\right)a_2(x, t) = -Ka_1(x, t)a_3(x, t) \quad (11)$$

$$\frac{\partial}{\partial t}a_3(x, t) = -Ka_1(x, t)a_2(x, t) \quad (12)$$

with  $K = \sqrt{w_1 w_{pl}}/2$ . You do not have to derive these equations. We will use a simulation to find a numerical solution for these equations using only first order approximation. There is a small wikipedia article I put on the recommended readings section (a kind of Euler Method for multivariate functions).

### 3.3 Calculations of Wave Parameters

Before you begin, you should open the file called "PUPCcodeFrame.m" in Matlab, and you will use this file to obtain the simulation. You will introduce the needed parameters, the equations and the other things you'll need here.

You must first write a code that calculates the initial amplitudes of  $a_1(t, x), a_2(t, x), a_3(t, x)$  of the waves, using the parameters of the waves I have described in the first part. These initial amplitudes are called  $a1.0, a2.0$  and  $a3.0$  in the code frame which is provided. Think about how you relate the intensity of each wave with the energy of it. Also calculate  $w_{pl}$ , the Langmuir Frequency  $w_3$ ,  $K$ , and the thermal velocity of the plasma  $v_{thm}$  as well, using the various quantities I have provided. Write your results into the code. As a general advice, as you write your code, you can take a look at section 3.7 Progression, Debugging and Plotting, which should help you with debugging and keeping your tasks organized.

1	2	3	4
$l_{pl} = 0.004m$	$l_{npl} = 0.001m$	$t_s = 500 * 10^{-15}s$	$t_p = 15 * 10^{-12}s$
$E_s = 70 * 10^{-6}J$	$E_p = 0.5J$	$\lambda_p = 351 * 10^{-9}m$	$\lambda_s = 400 * 10^{-9}m$
$D_{spot} = 150 * 10^{-6}$	$n_e = 1.1 * 10^{25}(m^{-3})$	$E_e = 8eV$	$t_{exp} = 16.5 * 10^{-12}s$

Table 1: Summary of parameters necessary to write simulation code.

### 3.4 Multivariate Functions: How to Represent the Data Over Time

To describe the behavior of the system, we will first need to learn how to use partial differential equations. We know that the system is described by the functions  $a_1, a_2$ , and  $a_3$ , specified for all times  $t$  and positions  $x$  in the interval under consideration. The position  $x$  naturally specifies the distance in the tube. It is best to choose a range of times which capture the laser pulse propagating across the tube. Let's consider an interval of time  $t_{exp} = 16.5 * 10^{-12}$ . Naturally,  $x$  falls in an interval spanned by the length  $L = l_{pl} + l_{npl}$  of the tube. Given the discrete nature of our analysis, we must choose how finely to chop up the temporal/spatial intervals. As an example, we could choose to chop up the  $x$  interval into  $N = 1500$  smaller intervals of length

$dx = L/N$ . We can also do something similar with the time interval, with some small interval of length  $dt$ . For making the implementation of the computation easier, we can choose  $dt = \frac{dx}{c}$ . Suppose we have  $m$  different  $x$  coordinates and  $n$  different time coordinates. We can therefore represent a function  $f(t, x)$  as a discretised matrix with entries  $f_{ij} = f(i dt, j dx)$ , where  $i = 0, \dots, m - 1$  and  $j = 0, \dots, n - 1$ . Thus, we can turn the multivariate functions  $a_1$ ,  $a_2$ , and  $a_3$ , into matrices. What is the easiest way to represent these spatial/temporal intervals,  $[0, dx, 2dx, \dots]$  and  $[0, dt, 2dt, \dots]$  in code? Implement your results.

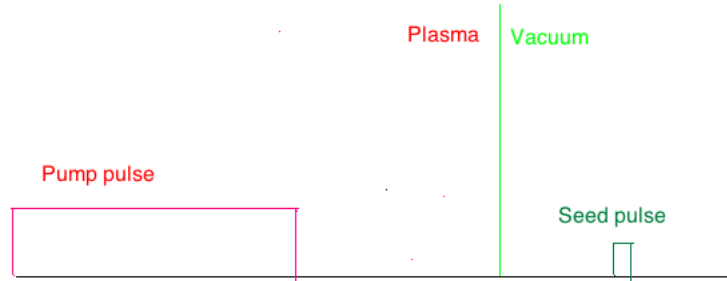


Figure 1: Schematic of the experimental setup.

### 3.5 Implementing Initial Conditions in the Simulation

Assume that initially the shape of  $a_1(0, x)$ , and  $a_2(0, x)$  have a rectangular profile, with initial constant amplitudes `a1_0` and `a2_0`, respectively. We make this assumption for simplicity of the code.<sup>1</sup> An example schematic of what this configuration looks like is shown in Fig. 1.

Try to define in the code the initial conditions for the three waves at all  $x$ 's and for  $t = 0$ . Say the left end of the tube is  $x = 0$  m and the right end of the tube is  $x = 0.005$  m. Remember that we have the pump and the seed coming from opposite directions and meeting initially at around  $x = 0.004$  m. You should think where the initial pulses should be in order to meet at the plasma vacuum interface (or at the place of your choice)

### 3.6 The Three Wave Equations and Solving for the Time Evolution of the Waves

Now, after creating the initial conditions, try to code equations (10), (11), and (12) to obtain the solution for the system. In particular, take the state of the system at a given time step (the solution for all  $x$ ), then solve for all  $x$  at the next time step.

There are a lot of intricacies regarding implementing this code for the solution over time. Think about how you need to write the partial differential equations in a discrete way, so that the resulting equations make sense even when “not much” happens in the system (the interaction pump-seed and transfer of energy have not started). You also need to remember that equations (10), (11), (12) were derived when all 3 waves are present. Take simpler cases and see if you understand what is happening. For the region lacking plasma, think about the energy exchange between the pump and the seed. Is there a Langmuir wave there? Think about how,  $a_1(t, x)$  and  $a_2(t, x)$ , and  $a_3(t, x)$  behave there. Remember that the seed comes from the right and the pump from the left, and that the Langmuir wave is a result of the interaction between the two, which sets the plasma in motion. Think what happens at  $x = 0$  m and  $x = 0.005$  m as you calculate with equations (10),(11),(12) what happens after each  $dt$ . Be careful at the plasma-vacuum interface so that the waves get transmitted in the right way. Also pay attention that the pump pulse will appear from the left of the simulation screen for some time as we can see from Figure 3, it moves to the right, till all the pump pulse passes trough the length we analyse. Notice that in Figure 4 there is no more pump laser pulse coming from the left.

<sup>1</sup> This rectangular profile does not meaningfully violate our earlier assumption that the envelopes of the potential are slowly varying; the phenomena we are interested in begin to happen when the seed profile  $a_2(x, t)$  is completely contained in the pump profile  $a_1(x, t)$ , and is not sensitive to the boundary of the pump or seed pulse profile. We can see this is true in Fig. 2.

### 3.7 Progression, Debugging, and Plotting

As you try to get the correct results for  $a_1(t, x)$ ,  $a_2(t, x)$ ,  $a_3(t, x)$  you can use the plotting function already in the code to plot each one of them as a function of  $x$ , at different moments  $t$ , and see their interaction, so that you might see what is wrong with the code, and fix it.

General word on a suggested progression with the code: A good idea is to write the code in small pieces, check that each part is right and then move to the next one. In general, if you try to code everything at once, you will end up having a lot of bugs, and it will take you a long time to find them. You could proceed in the following way: first, be sure that you have introduced all the correct parameters and that you have calculated  $a1_0$  and  $a2_0$  correctly. Then you can go ahead and set the initial values  $a_1(0, x)$ ,  $a_2(0, x)$ ,  $a_3(0, x)$  at the moment  $t=0$  and look on the plot to see if it looks right. Then try to make the pump and seed pulse propagate in the right directions without having any exchange of energy between them. How do they travel in this case? . As your pump pulse will come from the left for some time, you could think about how to make sure that you have the physical length of the pump pulse  $c \cdot t_p$  correct. The pulse will continue to appear on the plotting screen from the left for some amount of time, so that it has its total length  $c \cdot t_p$ . The seed pulse should also have the correct specified length,  $c \cdot t_s$ . After you see that the seed and pump pulse have the right length and travel in the right direction, you can try to directly code the interactions between the waves in equations (10), (11) and (12) using the Euler Method.

After you fix your code and are sure that everything is good, make some snapshots of the physical results (plots of  $a_1(t, x)$ ,  $a_2(t, x)$ ,  $a_3(t, x)$ ) using the function already in the code or one you made yourself. Also, display numerically the maximum intensity obtained for the seed, and the maximum  $a_2$  obtained in the process (you can use the function already in the code to do this as well).

### 3.8 Explanation of Amplification

**Note: even if you have not done the simulation you can attempt to answer these questions below.** After you have finished your simulation you should have produced some pictures similar to Figures 2, 3, and 4. If you have produced these images, then feel free to describe any details that you find in your own images rather than the images in this document. However, you can answer many of the questions below without generating your own simulation results.

**Questions** When is the Langmuir wave at a local maximum? What is the interaction between the waves at this point? Ask yourself if this result is what you expect. Explain in your own words what each wave is doing in the amplification process. How do the waves behave in the area with plasma and in the area with the vacuum? Briefly explain how each wave behaves and why.

#### 3.8.1 Creative Extensions of the Simulation

After you have finished the basic simulation, you can now vary some of the experimental parameters, simulate the results, and interpret the results in a physical manner. If you have not managed to get the simulation running try, to explain what you think should happen in each the following cases where we alter some parameter of the simulation:

Try varying electron density and see how the amplification gets modified. Explain what increasing or decreasing the electron density should do and explain why. Is this what you observe? Change the pump energy and seed energy and see how the amplification process changes. Attempt to explain qualitatively what you see when you vary the energy of each pulse and what you would expect to see. Vary  $t_p$  and  $t_s$  (the pulse time and seed time) and explain what you see or what you would expect to see.

Does the system process seem to have multiple stages of amplification? How does the length of the seed pulse change depending on how long it interacted with the pump pulse? Look at what happens when the two waves begin to interact and look at what happens after they interact for some time.

Feel free to take advantage of published resources when trying to answer these open ended questions. Finding other research to justify and support your claims is always beneficial when performing a scientific experiment.

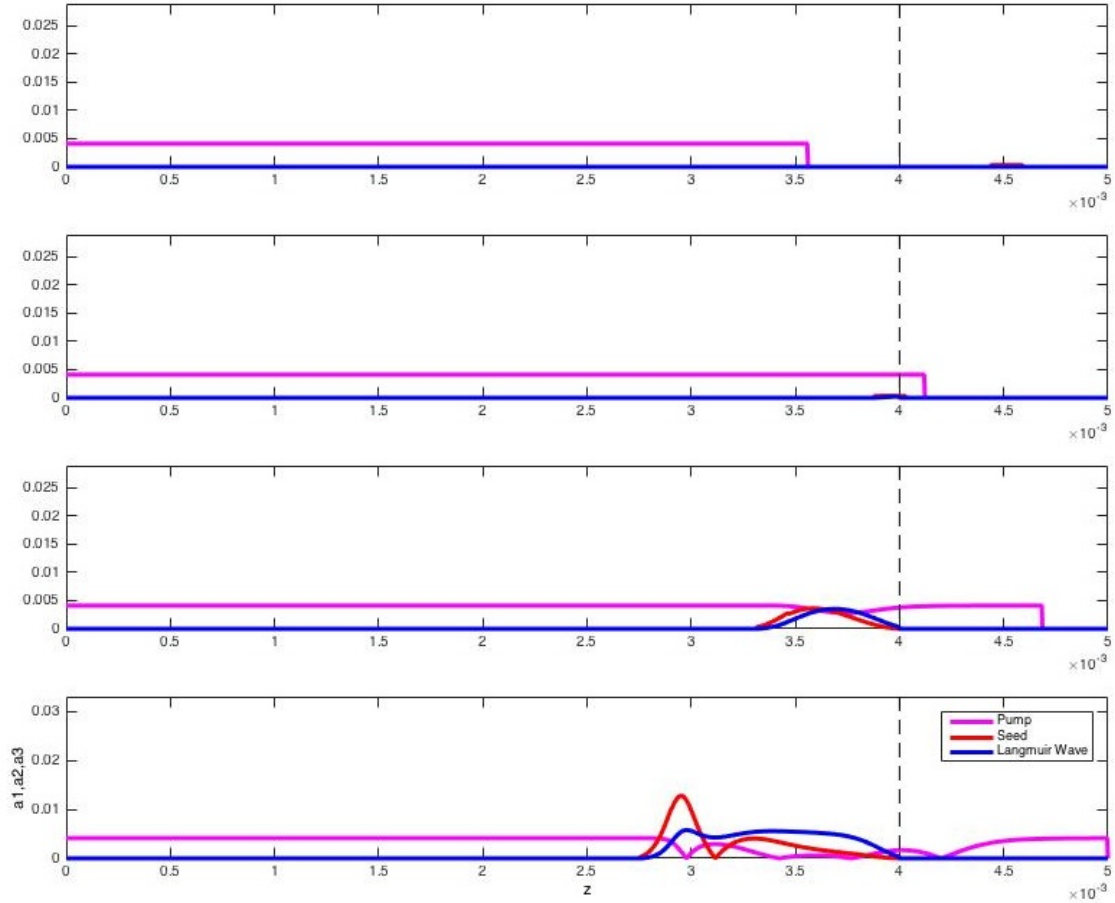


Figure 2: Initial stages of amplification.

### 3.9 Explanation of Figures and Tables

Figure 1 is a very simple scheme of the pump pulse, the seed pulse and the plasma vacuum interface. Figure 2 should help you answer section 3.6. Figure 3 presents different stages of amplification at different times, and Fig. 4 represents the amplification of the seed pulse at the end of the interaction. In addition, all the parameters you need to know for the simulation are stored in Table 1.

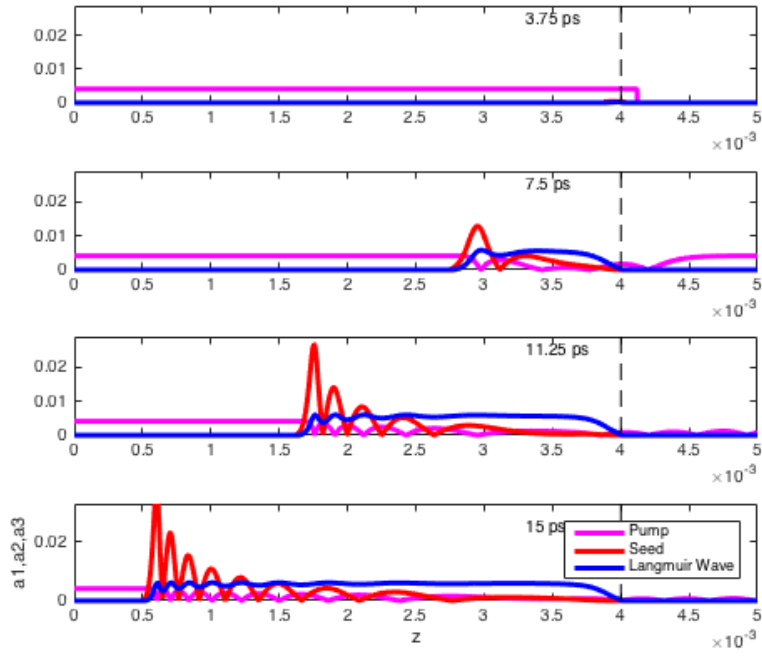


Figure 3: Intermediate stages of amplification.

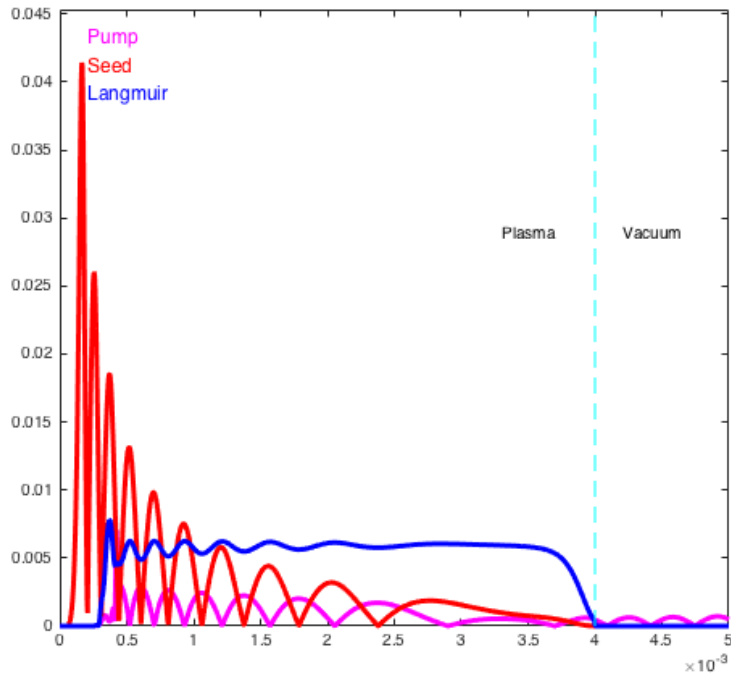


Figure 4: Final stage of amplification.